

## SOBOLEV OF THE EULER SCHOOL

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*On the occasion of the centenary of the birth of S. L. Sobolev*

ABSTRACT. This is a short overview of the origins of distribution theory as well as the life of Sergei Sobolev (1908–1989) and his contribution to the formation of the modern outlook of mathematics.

Sergei L'vovich Sobolev belongs to the Russian mathematical school and ranks among the scientists whose creativity has produced the major treasures of the world culture.

Mathematics studies the forms of reasoning. Generally speaking, differentiation discovers the trends of a process, and integration forecasts the future from trends. Mankind of the present day cannot be imagined without integration and differentiation. The differential and integral calculus was invented by Newton and Leibniz. The fluxions of Newton and the monads of Leibniz made these giants the forerunners of the classical analysis. Euler used the concepts by Newton and Leibniz to upbring and cultivate the new mathematics of variable quantities, while making quite a few phenomenal discoveries and creating his own inexhaustible collection of miraculous formulas and theorems. Mathematical analysis remained the calculus of Newton, Leibniz, and Euler for about two hundred years.

The classical calculus turned into the theory of distributions in the twentieth century. As the key objects of the modern analysis are ranked the integral in the sense of Lebesgue and the derivative in the sense of Sobolev which apply to the most general instances of interdependence that lie beyond the domains under the jurisdiction of the classical differentiation and integration. Lebesgue and Sobolev entered into history, suggesting the new approaches to the integral and derivative which expanded the sphere of influence and the scope of application of mathematics.

The historic figures and discoveries deserve the historical parallels and analysis. The gift of mathematics translates from teacher to student. The endless chain of alternating generations incarnates a mathematical tradition. Characterizing a scientific school, Luzin observed that “the elder school is more precious. Indeed, any school is the collections of the creative techniques, traditions, and narrations about the past and still living scientists as well as their manners of research and views of the object of research. These narrations are collected for ages but not intended for publication or revelation to those that seem undeserving. These narrations are treasures whose power is impossible to imagine or overrate . . . . If some analogy or

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comparison is welcome then the age of a school, together with the stock of its traditions and narrations, is nothing else but the energy of the school in implicit form.”<sup>1</sup> Sobolev belongs to the school that originated with Leonhard Euler (1707–1783).<sup>2</sup>

#### EULER AND RUSSIA

Man is a physical object and as such can be partly represented by his worldline in the 4-dimensional Minkowski space-time. “Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country,” Hilbert said at the Congress in Bologna in 1928.<sup>3</sup>

No state is a physical object. In space-time we may identify a country with the funnel of the worldlines of its inhabitants. The longest part of the worldline of Euler belongs to Russia. There is neither Russian nor Swiss mathematics. However, there is mathematics in Russia, there is a national mathematical tradition, and there is a national mathematical school. Born in Switzerland, Euler found his second homeland in Russia and is buried in the soil of St. Petersburg. Da Vinci of mathematics, he had become part and parcel of the Russian spirit. Our compatriots are proud to acknowledge Euler as the founder of the Russian mathematical school.

The efforts of Euler made Petersburg the mathematical capital of the eighteenth century. Daniel Bernoulli wrote to Euler: “I fail to convey to you quite properly how greedily they ask everywhere for the Petersburg memoirs.”<sup>4</sup> Implied were the celebrated *Commentarii Academiae Scientiarum Imperialis Petropolitanae* which became a leading scientific periodical of that epoch. The title of the journal changed many times and reads now as *Proceedings of the Russian Academy of Sciences (Mathematical Series)*. The journal of the Petersburg Academy of Sciences published 473 Euler’s articles which were printed successively during many years after his death up to 1830.

#### FROM OSTROGRADSKIĬ TO SOBOLEV

At the turn of the nineteenth century the center of mathematical thought shifted to France, the residence of Laplace, Poisson, Fourier, and Cauchy. The ideas of the new creators of mathematics were perceived by Ostrogradskiĭ who studied in Paris after he was deprived of his legitimate Graduation Diploma of Kharkov Imperial University. Cauchy appraised Ostrogradskiĭ in one of his papers of 1825 as a youngster gifted with a keen vision and rather knowledgeable in infinitesimal calculus.<sup>5</sup> The reputation of Ostrogradskiĭ in France, as well as a few memoirs submitted to the Academy of Sciences, led to the recognition of his merits in Russia. It was already in 1832 when Ostrogradskiĭ was elected as an ordinary academician in applied mathematics at the age of 32. Soon he became an undisputed leader of the Russian mathematical school.

Ostrogradskiĭ was fully aware of the importance of Euler to the science in Russia. He vigorously raised the question of publishing the legacy of Euler. In a relevant memo, Ostrogradskiĭ wrote: “Euler created the modern analysis, enriching it more than all his predecessors and making it the most powerful tool of human mind.”<sup>6</sup>

<sup>1</sup>From a private letter of Luzin. Cited from [1].

<sup>2</sup>Cp. [2] about Euler.

<sup>3</sup>Cited in [3, Ch. 21].

<sup>4</sup>Cp. [4, p. 101].

<sup>5</sup>Gnedenko in [4, p. 60] gave a reference to an article of 1901 by E. F. Sabinin.

<sup>6</sup>Cited from [4, pp. 101–102] where the reference is given to the Archive of the Academy of Sciences of the USSR, Fond 2, Description 1844, pp. 13–14.

The collection of 28 volumes was to be completed in 10 years, but the Academy had found no finances neither then nor ever after . . . .

N. D. Brashman, N. E. Zhukovskii, and S. A. Chaplygin are usually listed in the Moscow branch of the school of Ostrogradskii. The Petersburg branch included P. L. Chebyshev, A. M. Lyapunov, V. A. Steklov, and A. N. Krylov. Many other Russian mathematicians and mechanists were influenced by the research, teaching, and personality of Ostrogradskii.

Among the students of Chebyshev<sup>7</sup> we list A. N. Korkin and A. A. Markov who taught N. M. Günter, the future supervisor of the graduation thesis of Sobolev. As his second teacher, Sobolev acclaimed V. I. Smirnov, a student of V. A. Steklov who himself was supervised by A. M. Lyapunov. So is the brilliant chain of the scientific genealogy of Sobolev.<sup>8</sup>

Euler's archive belongs to Russia. However, the publication of the collected works of Euler was accomplished in Switzerland with the active participation of A. M. Lyapunov, A. N. Krylov, A. A. Markov, and V. I. Smirnov. The best minds of Russia strove to save the intellectual legacy of Euler. Smirnov rephrased the words of Goethe about Mozart as follows: "Euler will always remain a miracle beyond our ability to explain."<sup>9</sup> By now the 60 volumes of *Leonhardi Euleri Opera Omnia* are already published, and the whole collection of 72 volumes is planned to be completed this year.

#### MATHEMATICS OF RUSSIA IN THE 1930s

The great discoveries are the signposts of the inevitable which are not erected without efforts. Solving a problem presumes not only the statement of the problem but also some means and opportunities for solution. Necessity paves way through the impenetrable timberland of random events. Sobolev's contributions belong to the epoch of tremendous breakthroughs in the world science. The twentieth century is rightfully called the age of freedom. The development of the institutions of democracy was accompanied with the liberation of all aspects of the mental life of mankind. Mathematics has revealed its essence of the science of the forms of free thinking. Freedom is a historical concept reflecting the manner of resolving the clashes between the individuals loose in diversity and the tight bonds of their collective coexistence. The historical entourage is an indispensable ingredient of any triumph and any tragedy.

Pondering over his achievements in 1957, Sobolev noticed:<sup>10</sup>

In the study of the various problems of finding the functions that satisfy some partial differential equations, it turned out fruitful to use some class of the functions that fail to possess the continuous derivatives of appropriate order everywhere but serve in a sense as the limits of the genuine solutions of the equations. Naturally, we seek for these generalized solutions in various function spaces, sometimes complete and sometimes to be especially completed with the aid of new "ideal" elements.

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<sup>7</sup>About Chebyshev cp. [5].

<sup>8</sup>See an overview of the history of the Petersburg–Leningrad mathematical school in [6]. A few details of the green years of Sobolev are collected in [7].

<sup>9</sup>Cp. [8, p. 54].

<sup>10</sup>Cited from [9, p. 596] which is a reprint of an article in *Vesnik Drushtva Matematichara i Fizichara Narodne Republike Srbije (Jugoslavija)* also known as *Bulletin de la Société des Mathématiciens et Physiciens de la R. P. de Serbie (Yougoslavie)*, **9**, 215–244 (1957) (Zbl 0138.34503).

Science has traveled from an individual solution to studying the function spaces, operators between the spaces, and those elements that are solutions.

The problem arises of importance in its own right of the conditions for these generalized solutions to be classical.

We see that Sobolev distinguished the close connection of his theory with the Hilbert idea of socializing mathematical problems. Hilbert's methodology rested on the Cantorian set theory.

The idea of revising the concept of solution of a differential equation was in the mathematical air of the early twentieth century. The interest of Sobolev in this topic is undoubtedly due to Günter. In the obituary by Sobolev and Smirnov, they emphasized the role of Günter in propounding the Lebesgue idea of the necessity of a new approach to the equations of mathematical physics on the basis of the theory of set functions.<sup>11</sup>

Sobolev learned the ideas of functional analysis in the seminar headed by Smirnov. The program of the seminar included the study of the classical book by J. von Neumann on the mathematical foundations of quantum mechanics. Von Neumann sharply criticized the approach by Dirac:

Die "uneigentlichen" Gebilde (wie  $(\delta(x), \delta'(x), \dots)$ ) spielen in ihnen eine entscheidende Rolle — sie liegen außerhalb des Rahmens der allgemein üblichen mathematischen Methoden ...<sup>12</sup>

The "improper" functions (such as  $\delta(x), \delta'(x), \dots$ ) play a decisive role in this development —they lie beyond the scope of mathematical methods generally used ...<sup>13</sup>

The ideas of von Neumann attracted another participant of the Smirnov seminar, Leonid Kantorovich who became a friend of Sobolev in their university years. In 1935 Kantorovich published two articles in *Doklady AN SSSR* 4 (1935) which were devoted to introducing "certain new functions, 'ideal functions' that would not be functions in the strict sense of the word." His articles were written in the spirit of Friedrichs and contained the distributional derivatives of periodic tempered distributions.<sup>14</sup> In 1991 Israel Gelfand appraised these articles as follows: "In essence, Leonid Vital'evich was the first who understood the importance of generalized functions and wrote about the matter much earlier than Laurent Schwartz."<sup>15</sup>

It seems absolutely improbable that Sobolev and Kantorovich, old cronies and members of the same seminar, could be unaware of the articles by one another which addressed the same topic. However, neither of the two had ever mentioned the episode in future. It becomes clear that the 1930s were the years of a temporary detachment between Sobolev and Kantorovich who cultivated a warm and cordial friendship up to their last days. The political events of the 1930s in the mathematical circles of Leningrad and Moscow seem helpful in understanding the predicament.

<sup>11</sup>In particular, cp. [10]. The original book by Günter appeared in French in 1934. The English translation by John R. Schulenberger was published in 1967 by the Frederick Ungar Publishing Co. in New York (Zbl 0164.41901).

<sup>12</sup>Cp. [11, p. 15]. Von Neumann remarked earlier that "DIRAC fingierte trotzdem die Existenz einer solchen Funktion" (cp. [11, p. 14]).

<sup>13</sup>This translation by R. Beyer was published by the Princeton University Press in 1957.

<sup>14</sup>Cp. [12, 13].

<sup>15</sup>Cp. [14, p. 162]. Gelfand's article appeared firstly in the periodical collection of the Sobolev Institute of Mathematics—*Optimizatsiya* 50(67) (1991), 131–134. There is a very rough English translation of the article in the first volume of the *Selected Works* of Kantorovich which was printed in 1996. Sobolev's article "The Cauchy problem in the space of functionals" was published in *Doklady AN SSSR* 3 (1935) and reprinted in [9, pp. 11–13].

The “Leningrad mathematical front” was launched against the old mathematical professorate of the Northern capital of Russia. Günter, leading the Petrograd Mathematical Society from its reestablishment in 1920, was chosen as the main target of the offensive. Günter was not only accused in all instances of misconduct, idealism, and neglect of praxis but also branded as a “reactionary in social life” and “conservative in science.” The “Declaration of the Initiative Group for Reorganization of the Leningrad Physical and Mathematical Society” as of March 10, 1931, containing dreadful accusations against Günter was endorsed by 13 persons, among them I. M. Vinogradov, B. N. Delaunay, L. V. Kantorovich, and G. M. Fikhtengolts. Günter was forced to resign as the chair of a department and had no choice but writing a letter of repentance which was nonetheless condemned by the “mathematician-materialists.” Steklov, who had died in 1926, was ranked as a member of the band of idealists either.<sup>16</sup> Sobolev and Smirnov must be commended for abstaining from the public persecution of their teachers.<sup>17</sup> The antidote transpires in the definite affinity of the scientific views of the teachers and the students.

The situation in the mathematical community differed slightly from the routine of the epoch. The old professorate was pursued in Moscow either.<sup>18</sup> The Muscovites attempted to involve Kantorovich in their quandaries, since he was appraised among the top specialists in the descriptive theory of sets and functions. Kantorovich refrained from any offensive against Luzin, whereas Sobolev became an active member of the emergency Commission of the Academy of Sciences of the USSR on the “case of Academician Luzin.”<sup>19</sup>

Omnipresent was the tragedy of mathematics in Russia. So were the triumphs.

#### SOBOLEV AND THE A-BOMB

*Homo Sapiens* reveals himself as *Homo Creatoris*. The power of man is his capability of creating and transferring intangible valuables. Mathematics saves the ancient technologies of impeccable intellectual conjurations. The art and science of provable calculuses, mathematics resides at the epicenter of culture. The freedom of reasoning is the *sine qua non* of the personal liberty of a human being. Mathematics in the foundations of mentality becomes the guarantee of freedom. The creative contributions of Euler as well as his best descendants exhibit uncountably many supreme examples. The fate of Sobolev made no exclusion.

In the twentieth century mankind came to the edge of the frontiers of its safe and serene existence, exhibiting the inability of halting the instigators of the First and Second World Wars. The weapon of deterrence arose as a warrant of freedom. The invention and production of the A-bomb in the USA and Russia demonstrate the tremendous power of science, the last resort of the survival of mankind. Mathematicians may be proud of the valor of their colleagues in these exploits. Von Neumann and Ulam participated in the Manhattan project. Sobolev and Kantorovich were involved in the Soviet project “Enormous.”<sup>20</sup>

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<sup>16</sup>The “Declaration” and other documents of the “Leningrad mathematical front” are collected in the booklet [15].

<sup>17</sup>Also, Smirnov had his black mark as listed among the right-wing peacemakers and advocates of Günter [15, pp. 10, 33].

<sup>18</sup>Cp. [16] for relevant references.

<sup>19</sup>The historical details and shorthand minutes of the meetings of the Committee are collected in [17].

<sup>20</sup>Transliterated in Russian like “Énormoz.” This code name was used in the operative correspondence of the intelligence services of the USSR.

Most documents are declassified and published about the making of nuclear weapons, and so we may feel the tension of the heroic epoch.

The start of the atomic project in this country is traditionally marked with Directive No. 2352ss<sup>21</sup> of the SDC<sup>22</sup> which was entitled “Organization of the Works on Uranium” and dated September 24, 1942.<sup>23</sup> A few months later on February 1943, the SDC decided to organize Laboratory No. 2 of the Academy of Science of the USSR for studying the nuclear energy. I. V. Kurchatov was entrusted with the supervision of the Laboratory as well as the management of all works related to the atomic problem. Sobolev was soon appointed one of the deputies of Kurchatov and joined the group of I. K. Kikoin which studied the problem of enriching uranium with cascades of diffusive membranes for isotope separation.

The Special Folder<sup>24</sup> saves the report by Kurchatov and Kikoin as of August 1945. The preamble of this document reads:

The work on utilizing the internuclear energy started in the USSR in 1943 when Laboratory No. 2 was arranged in the Academy of Sciences under the leadership of Academician Kurchatov I. V.

Since the Laboratory has no premises, facilities, cadres, and uranium, the work was reduced to analyzing the secret materials about the investigations of the foreign scientists in the uranium problem as well as checking these data by calculation and performing of a few experiments.

In the second half of 1944 and [in] the beginning of 1945, Laboratory No. 2 had received support by a decision of the SDCO with premises, facilities, materials, and cadres, which enables the Laboratory to launch its own research.

A series of institutions as well as design and construction organizations of the USSR were assigned to work by the program of Laboratory No. 2 (including the Radium, Physical, and Energy Institutes of the Academy of Sciences of the USSR, the All-Union Institute of Mineral Resources, the State Rare Metal Institute, the State SRI<sup>25</sup>-42, etc.).

As regards the methods for acquiring the atomic explosives (uranium-235 and plutonium-239) which are known abroad, namely, the method of the “uranium-graphite boiler,” the method of the “uranium-deuterium boiler,”<sup>26</sup> the diffusion method, and the magnetic method, the top officials of Laboratory No. 2 (Academicians Kurchatov and Sobolev together with Corresponding Members of the Academy of Sciences Kikoin and Voznesenskii) opine that the Laboratory has already the data on the first three of the methods which is enough for designing and erecting the facilities.<sup>27</sup>

It was already in 1946 that the first gaseous compressors were produced and put into the serial production. The tests began of enriching uranium hexafluoride. The work required solving incredibly many versatile scientific, technological, and managerial problems which became the main busyness of Sobolev for many years. It suffices to give the list of problems from a memo for L. P. Beriia as of August

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<sup>21</sup>The letters “ss” abbreviate the Russian for “top secret.”

<sup>22</sup>This is the acronym of the State Defence Committee of the USSR. Another acronym was SDCO.

<sup>23</sup>The original was not signed by the Chairman of the SDC I. V. Stalin who had a habit of endorsing the front cover of the whole folder with a pile of documents. The appended mailing list indicates that the full text of the Directive was forwarded to V. M. Molotov, S. V. Kaftanov, A. F. Ioffe, V. L. Komarov, and Ya. E. Chadaev.

<sup>24</sup>In those days the term “special folder” was also a formal top secrecy stamp.

<sup>25</sup>This is the acronym for the state research institute.

<sup>26</sup>The term “heavy water” is used in the original. The locution “boiler” stands for “pile” and “reactor.”

<sup>27</sup>The whole document is presented in [18, p. 307]. There is a handwritten note by Stalin: “Due for reading.”

15, 1946:<sup>28</sup>

1. Choice of the general scheme of the technological process of the industrial separation plant.
2. Raw materials.
3. The problem of filters.
4. Compressors.
5. The problem of the pressurization (hermetic sealing) of compressors and lubrication.
6. The problem of corrosive materials in uranium hexafluoride.
7. Analysis of the enrichment of the light isotope.
8. The problem of control and automation.

Sobolev joined the group for plutonium-239 and the group for uranium-235.<sup>29</sup> He organized and coordinated the work of the staff of calculators, solved the problem of control of the industrial isotope separation, and was responsible for minimizing the losses of production. His role in the atomic project became more important. In February of 1947 Kurchatov wrote to Beriya:

By now Academician S. L. Sobolev was acquainted only with the documents of Bureau No. 2 which are related to the diffusion method. In regard of his appointment to the position of the Deputy Principal of Laboratory No. 2 of the Academy of Sciences of the USSR, I ask your permission to acquaint Academician Sobolev S. L. with the documents of Bureau No. 2 concerning all aspects of the problem.<sup>30</sup>

The test of the Joe-1<sup>31</sup> took place near Semipalatinsk at 8 a.m. local time on August 29, 1949. Exactly two months later more than eight hundred staff members of the atomic project were decorated with various state orders. Sobolev was awarded with the Order of Lenin. It was in the mid 1949 that Laboratory No. 2 was renamed to become the Laboratory of Measuring Tools of the Academy of Sciences, abbreviated as LIPAN in Russian. The efforts of Kikoin and Sobolev were focused on the manufacturing program of the diffusion plant. One of the items of Decree No. 5472-2086ss/op<sup>32</sup> of the Council of Ministers of the USSR as of December 1, 1949 reads:

Entrust Comrade Sobolev S. L. (Deputy Principal of Laboratory No. 2 of the Academy of Sciences of the USSR) with the management of the theoretical calculation section of the Central Laboratory of Combine No. 813,<sup>33</sup> on requesting that he be on duty at the combine for at least 50% of the whole working hours (on consent of Comrade Kurchatov I. V.).<sup>34</sup>

In the LIPAN Sobolev wrote the main book of his life, *Some Applications of Functional Analysis in Mathematical Physics*.<sup>35</sup>

The atomic project enriched the scientific and personal potential of Sobolev. Computational mathematics occupied a prominent place in his creative activities up to his last days. From 1952 to 1960 he held the chair of the department of computational mathematics at Lomonosov State University. Later in Siberia, Sobolev

<sup>28</sup>Cp. [18, p. 567].

<sup>29</sup>See [18, p. 386].

<sup>30</sup>Cited from [19, p. 432]. This top secret document was handwritten in a sole copy and bears the resolution by Beriya: "Agreed. L. Beriya. 21/11-47."

<sup>31</sup>Joe-1 was the English nickname for the Soviet A-bomb No. 1. The Russian codename was RDS-1.

<sup>32</sup>The letters "op" imply the stamp "special folder" in Russian.

<sup>33</sup>Now it is the Ural Electromechanical Plant in Novouralsk, formerly known as Sverdlovsk-44.

<sup>34</sup>Cp. [19, pp. 363-364].

<sup>35</sup>Published in 1950 by Leningrad State University, reprinted in 1962 by the Siberian Division of the Academy of Sciences of the USSR in Novosibirsk, and translated into English by the American Mathematical Society in 1963. The third Russian edition was printed by the Nauka Publishers in 1988. The book was reproduced by the AMS in 1991, and a new printing is scheduled this year.

propounded the theory of cubature formulas which is wondrous in the beauty of its universality. Sobolev synthesized the ideas of the classical approximative methods and distribution theory. Sobolev suggested that calculations on a mesh should be considered as some integrals involving distributions. This was done within his deep belief in the indissoluble ties between functional analysis and the theory of computations.

The work in the LIPAN added many bright colors to Sobolev's views of mathematics. Those years brought to him the understanding that of importance in many cases is the actual presentation of a reasonable solution on the appointed time rather than the abstract problem of existence of a solution.

The outstanding importance for the history of science in this country must be allotted to the Sobolev talks at the All-Union Conference on the Philosophical Problems of Natural Sciences in October 1958. Elaborating and maintaining the theses of a joint report with A. A. Lyapunov,<sup>36</sup> Sobolev guarded science from the interference of the prevalent ideology and defended the ideas of cybernetics and genetics, sharply criticizing the rigmarole of neolamarkism.<sup>37</sup> The report claimed in particular that "no scientist would ever propound the thesis of the adaptive heredity or directed evolution independent of selection" [20, p. 252]. In his closing talk, Sobolev said<sup>38</sup>:

... cybernetics is not an idealistic science since it studies facts, and the facts are neither materialistic nor idealistic ... It is impossible to divide physics into materialistic physics and idealistic physics. It is impossible to declare that this A-bomb is idealistic whereas that A-bomb is materialistic, or this particle accelerator is idealistic whereas that one is materialistic. None of these ever exists. The main road of physics is the road of a rigorous science. There might exist various philosophical views, but we must not classify as materialism or idealism the facts and theories that led to the greatest achievements of the modern physics which we observe. Exactly the same applies to cybernetics ...

The proceedings of the conference were printed in many copies,<sup>39</sup> demonstrating to the academic community of this country that the defence of science can be conducted not only in the submissive form of personal or collective letters to the Central Committee of the Communist Party of the USSR.

The civic courage of Sobolev in safeguarding the new ideas of genetics, cybernetics, and mathematical economics in the postwar years of the offensive of the obscurantists of "Marxism" ranks alongside his participation in the "Enormous" project and cultivation of the scientifically virgin lands of Siberia.

The contribution of Sobolev to the making of nuclear weapons is acknowledged and marked not only with the title of the Hero of the Socialist Labor but also the eternal gratitude of the people of this country to the famous and anonymous saviors of the freedom of the homeland.

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<sup>36</sup>Printed in [20, pp. 237–260].

<sup>37</sup>Everyone understood that the object of criticism was T. D. Lysenko.

<sup>38</sup>Cp. [20, p. 572].

<sup>39</sup>The book was endorsed for printing on October 10, 1959. It is worth recalling that N. S. Khrushchëv made a speech at the Plenum of the Central Committee of the Communist Party of the USSR on June 29, 1959 in which he praised Lysenko, rebuked N. P. Dubinin for the lack of scientific contribution, and reprimanded the leadership of the Siberian Division for appointing Dubinin as the director of the Institute of Cytology and Genetics of the Siberian Division of the Academy of Sciences of the USSR (cp. [21, pp. 192–199]).



## NEW DERIVATION—NEW CALCULUS

Sobolev's contributions are connected with the reconsideration of the concept of solution to a differential equation. He suggested that the Cauchy problem be solved in the dual space, the space of functionals, which means the rejection of the classical view that any solution of any differential equation presents a function. Sobolev proposed to assume that a differential equation is solved provided that all integral characteristics are available of the behavior of the process under study. Moreover, the solution as a function of time may fail to exist at all rather than stay unknown for us temporarily. In actuality, science has acquired a new understanding of the key principles of prognosis.

It was as long ago as in 1755 that Euler gave the universal definition of function which was perceived as the most general and perfect. In his celebrated course in differential calculus, Euler wrote:<sup>40</sup>

If, however, some quantities depend on others in such a way that if the latter are changed the former undergo changes themselves then the former quantities are called functions of the latter quantities. This is a very comprehensive notion and comprises in itself all the modes through which one quantity can be determined by others. If, therefore,  $x$  denotes a variable quantity then all the quantities which depend on  $x$  in any manner whatever or are determined by it are called its functions.

The generalized derivatives in the Sobolev sense do not obey the Eulerian definition of function. Differentiation by Sobolev implies the new conception of interrelation between mathematical quantities. A generalized function is determined implicitly from the integral characteristics of its action on each representative of some class of test functions that was chosen in advance.

The discoveries by Newton and Leibniz summarized the centenary-old prehistory of differential and integral calculus,<sup>41</sup> opening the new areas of research. The achievements of Lebesgue and Sobolev continued the contemplations of their glorious predecessors and paved the turnpike for the present-day mathematicians.<sup>42</sup>

Sobolev was among the pioneers of application of functional analysis in mathematical physics, propounding his theory in 1935. In the articles by Laurent Schwartz<sup>43</sup> who came to the similar ideas a decade later, the new calculus became comprehensible and accessible for everyone in the elegant, powerful and rather transparent form of the theory of distributions which has utilized many progressive ideas of algebra, geometry, and topology.

Lavish was the Sobolev appraisal of the contribution of Schwartz into the elaboration of the technique of the Fourier transform for distributions:<sup>44</sup>

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<sup>40</sup>Cp. [22, p. 38] and [23, pp. 72–73].

<sup>41</sup>Noneuropan roots of analysis are still uncharted. About Seki Takakazu Kōwa and Mādhava of Sangamagrama see [24, p. 310], [25].

<sup>42</sup>Consult [24] about the prehistory of distributions. The famous quandary between Euler and d'Alembert about the vibrating string was a harbinger of search into the abstractions of the concept of a solution to a differential equation (cp. [26, pp. 15–24] and [27]). Euler's liberal handling of divergent series have reflected the flashes of the future theory of distributions (cp. [2, pp. 187–188]).

<sup>43</sup>Schwartz's views of the discovery of distribution theory are presented in his autobiography [28]. A few relevant references are given in [29].

<sup>44</sup>Sobolev dated the theory of generalized functions from his paper of 1935 and wrote: "The theory of generalized functions was further developed by L. Schwartz [21] who has in particular considered and studied the Fourier transform of a generalized function" (cp. [30, p. 355]). This is a curious misprint: the correct reference to Schwartz's two-volume set should be [47].

The generalized functions, in much the same way as the ordinary functions, can be subjected to the Fourier transform. We may say even more: In the classical calculus, the Fourier transform was confronted with many considerable difficulties such as the divergence of integrals, the impossibility of interpreting the resultant infinite expressions in a definite sense, and so on. The theory of generalized functions eliminated most of these difficulties and made the Fourier transform a powerful tool of analysis.<sup>45</sup>

The differential calculus of the seventeenth century is inseparable from the general views of the classical mechanics. Distribution theory is tied with the mechanics of quanta.

We must emphasize that quantum mechanics is not a plain generalization of the mechanics of classics. Quantum mechanics presents the scientific outlook that bases on the new laws of thought. The classical determinism and continuity swapped placed with quantization and uncertainty. It was in the twentieth century that mankind raised to a completely new comprehension of the processes of nature.

Similar is the situation with the modern mathematical theories. The logic of these days is not a generalization of the logic of Aristotle. Banach space geometry is not an abstraction of the Euclidean plane geometry. Distribution theory, reigning as the calculus of today, has drastically changed the whole technology of the mathematical description of physical processes by means of differential equations.

Sobolev heard the call of future and bequeathed his spaces to mankind.<sup>46</sup> His discoveries triggered many revolutionary changes in mathematics whose progress we are happy to observe and follow.

The terminal series of Sobolev's mathematical articles was devoted to the subtle properties of the roots of the Euler polynomials . . . .

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<sup>45</sup>Cp. [30, p. 415].

<sup>46</sup>In an untitled poem with the first line “It is not seemly to be famous” as of 1956, Boris Pasternak wrote (as rendered in English by Lydia Pasternak Slater, cp. [31, p. 255]):

Try not to live as a pretender,  
But so to manage your affairs  
That you are loved by wide expanses,  
And hear the call of future years.

The verbatim translation of the Russian original of the third line of the above excerpt contains the marvelous expression “love of space.” So, the above verses might be rendered as follows:

Avoid conceit and self-pretense,  
But pace up life and make your soul  
Attract to you the love of space  
And read the distant future call.

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